Table of Contents

1. Fourier Transforms 1
1.1 The Fourier Transform 1
Example 1: The *rect* Function 2
Example 2: The *delta* Function 3
Example 3: The Inverse Fourier Transform 6
Example 4: Another Look at the Delta Function 7
1.2 Fourier Optics 8
Example 5: The Fourier Transform of a Slit 10

2. Scaling and Shifting Theorems 13
2.1 Scaling Theorem 13
Example 6: The Fourier Transform of $\text{rect}(t/N)$ 14
2.2 Time Shifting Theorem 14
Example 7: The Fourier Transform of $\text{rect}\left((\text{t-t}_0)/\text{a}\right)$ 14
Example 8: The Fourier Transform of a Two-Pulse Signal 16
Example 9: The Fourier Transform of a Three-Pulse Signal 17
Example 10: The Fourier Transform of an *N*-Pulse Signal 19
Example 11: The Fourier Transform of a Delta Function 21
Example 12: The Fourier Transform of a Pulse Train 22
Example 13: The Fourier Transform of an Impulse Train 23
2.3 Frequency Shifting Theorem 24
Example 14: The Fourier Transform of $\cos(2\pi f_0 t)$ 25
Example 15: The Fourier Transform of $g(t)\cos 2\pi f_0 t$ 25
Example 16: The Fourier Transform of $\text{rect}(t/a)\cos 2\pi f_0 t$ 25
2.4 Reciprocal Property of Fourier Transforms 26
Example 17: The Fourier Transform of $\text{sinc}(t)$ 27

3. Convolution and Correlation 28
3.1 Convolution 28
Example 18: Convolution of Two *rect* Functions 29
Example 19: Convolution of Multiple-Pulse Signals 31
3.2 Cross-Correlation 31
Example 20: Cross Correlation of 2- and 3-Pulse Signals 33

Example 21: Cross Correlation of 3- and 2-Pulse Signals 34
3.3 Autocorrelation 35
Example 22: Autocorrelation vs. Convolution 36
3.4 Convolution and Multiplication 37
Example 23: Amplitude Modulation 38
3.5 Parseval's Theorem 40
3.6 Autocorrelation and Power Spectrum 41

4. Periodic Functions and Fourier Series 43
Example 24: Convolution with a Delta Function 43
Example 25: The Fourier Transform of *N* Pulses Using Delta Functions 44
Example 26: Complex Fourier Series of a Periodic Signal 47
Example 27: Complex Fourier Series of a Pulse Train 50
Example 28: Trigonometric Fourier Series of a Periodic Signal 51
Example 29: The Trigonometric Fourier Series of a Pulse Train 52
Example 30: MATLAB Solution for Plotting the Trigonometric Fourier Series of a Pulse Train 53

5. The Discrete-Time Fourier Transform 56
Example 31: The Fourier Transform of Sampled Data 56
Example 32: The Whittaker-Shannon Sampling Theorem 60
Example 33: The Discrete-Time Fourier Transform 63
Example 34: MATLAB Calculation of the DTFT 65

6. The Discrete Fourier Transform 68
Example 35: Making a Sampled Input Periodic 68
Example 36: The Discrete Fourier Transform 70
Example 37: Calculating the Discrete Fourier Transform 71
Example 38: The DFT of a Triangle Function 73
Example 39: The Inverse Discrete Fourier Transform 77
Example 40: The DFT Basis Functions 78

7. The Fast Fourier Transform 81
Example 41: Speeding Up the Calculation of the DFT 81
Example 42: The Fast Fourier Transform Algorithm 83

8. MATLAB Examples of the FFT 90
Example 43: FFT of a Triangle Function 90
Example 44: FFT of Cosine and Sine Waves 95
Example 45: FFT of Multiple Cosine Waves 105
Example 46: FFT of a Rectangular Cosine Wave 108
Example 47: FFT of a Gaussian Cosine Wave 110
Example 48: FFT of a Major C Chord 114
Example 49: FFT of an Amplitude Modulated Signal 116
Example 50: FFT of a Frequency Modulated Signal 119
Example 51: FFT of a Binary Phase Shift Keying Signal 122
Example 52: FFT of a Quadrature Phase Shift Keying Signal 125

9. The Discrete Cosine Transform 128
Example 53: The DCT Basis Functions 129
Example 54: Calculating the Discrete Cosine Transform 131
Example 55: Calculating the DCT from the FFT 133
Example 56: The DCT of Image Data 136
Example 57: Searching for an Inverse DCT 137
Example 58: A Direct Inverse DCT 140
Example 59: Calculating the Inverse DCT from the IFFT 142
Example 60: Two-Dimensional DFT 147
Example 61: Two-Dimensional DCT 150
Example 62: Two-Dimensional Inverse DCT 152
Example 63: Using the DCT in JPEG Encoding 154

Appendix A – Fraunhofer Diffraction 167
Appendix B – Fourier Transform of a Triangle Function 172
Appendix C – Fourier Transform of a Gaussian Function 175

Index